$\frac{}{(t-a)^{n+1}}$

p.G.-sem-2nd paper-VI, unit-1st complex integral or Cauchy's integral

(1) Theorem 7 :→

State and prove Liouville's theorem 14 मंगल

Statement :→ If an entire function is bounded for all values of z, then it is constant.

Or

If a function f(z) is analytic for finite value of z, and is bounded then f(z) is constant.

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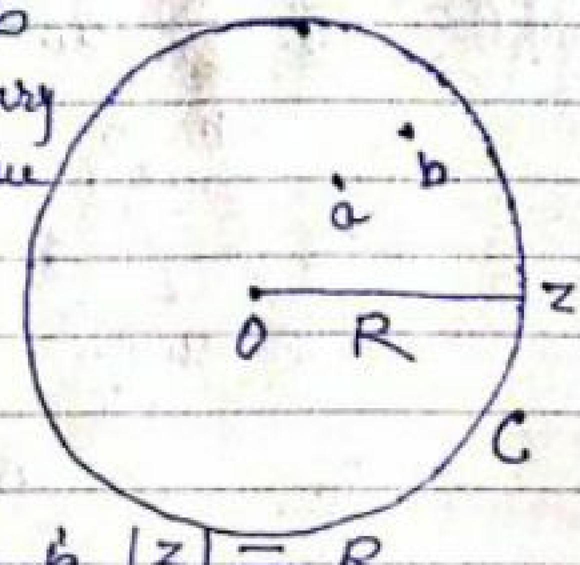
Proof:→ Let a and b be any two arbitrary distinct points in z-plane and let C be a large circle with centre $z=0$ and radius R so that it encloses a and b.



The equation of the circle is $|z| = R$

$$\therefore z = Re^{i\theta}, \quad dz = iRe^{i\theta}d\theta$$

$$|dz| = R\,d\theta$$

Since $f(z)$ is bounded for all C

$$\therefore |f(z)| \leq M \;\forall z \text{ where } M>0$$

By Cauchy's Integral formula

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$$f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)\,dz}{z-a}, \quad f(b) = \frac{1}{2\pi i}\int_C \frac{f(z)\,dz}{z-b}$$

$$\therefore f(a) - f(b) = \frac{1}{2\pi i}\int_C \left(\frac{1}{z-a} - \frac{1}{z-b}\right) f(z)\,dz$$

$$= \frac{a-b}{2\pi i}\int_C \frac{f(z)\,dz}{(z-a)(z-b)}$$

$$\therefore |f(a) - f(b)| \leq \frac{|a-b|}{2\pi}\int_C \frac{|f(z)||dz|}{(|z|-|a|)(|z|-|b|)}$$

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(3)

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or $\left| f(a) - f(b) \right| \le \frac{|a-b|}{2\pi} \frac{M 2\pi R}{(R-|a|)(R-|b|)}$

$$\le \frac{M\,|a-b|\,\frac{1}{R}}{\left(1-\frac{|a|}{R}\right)\left(1-\frac{|b|}{R}\right)} \to 0 \quad \text{as } R \to 0$$

$\therefore f(a) - f(b) = 0$ or $f(a) = f(b)$

which shows that $f(z)$ is constant

Proved